

Natural SUSY from SU(5) Orbifold GUT

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ABSTRACT: We propose a realistic 5D orbifold GUT model that can reduce to natural (or radiative natural) supersymmetry as the low energy effective theory. Supersymmetry as well as gauge symmetry are broken by the twist boundary conditions. We find that it is non-trivial to introduce other flavor symmetry other than the $SU(2)_R$ R-symmetry. We ameliorate the tension between the small number of free parameters and the successful electroweak symmetry breaking by introducing non-minimal Kahler potentials. A large trilinear term A_t , which is necessary to give a 125 GeV Higgs boson, is naturally predicted in our scenario. A scan under current experimental constraints shows that our model can indeed realize natural (or radiative natural) supersymmetry.

KEYWORDS: SUSY, GUT, Naturalness.

Contents

1. Introduction	1
2. A brief review of Grand Unified Theories	2
3. SUSY soft masses from Scherk-Schwarz mechanism	4
4. Natural SUSY with a large A_t term	10
5. Viable parameter space	13
6. Conclusions	17

1. Introduction

Both the ATLAS [1] and CMS [2] collaborations have now established the existence of a 125 GeV Higgs-like boson from the combined 7 TeV and 8 TeV LHC data. Although the data are so far in agreement with the Standard Model (SM) prediction, they can also be accommodated by many new physics frameworks, among which a particularly interesting and widely studied scenario is supersymmetry (SUSY). An interesting observation is that the mass value of the observed Higgs-like boson just falls within the narrow range of 115 – 135 GeV predicted by the Minimal Supersymmetric Standard Model (MSSM).

SUSY naturally solves the gauge hierarchy problem of the SM and at the same time provides a viable dark matter candidate. In SUSY the unification of the three gauge couplings of $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ at about 2×10^{16} GeV [3] strongly suggests the existence of Grand Unified Theories (GUTs). In addition, the SUSY GUTs such as $SU(5)$ [4] or $SO(10)$ [5] models give us deep insights into the other SM problems such as the emergence of the fundamental forces, the assignment and quantization of their charges as well as the fermion masses and mixings. Although SUSY GUTs are attractive, it is challenging to test them at the Large Hadron Collider (LHC) or the future International Linear Collider (ILC).

It is well known that some problems like the doublet-triplet splitting always exist in many GUT models. One elegant way to solve this problem is to put the GUT gauge group into the five-dimensional (5D) bulk and the GUT symmetry is broken by boundary conditions, for example, by orbifold projection. Orbifold GUT models for $SU(5)$ were proposed in [6] and then widely studied in [7, 8, 9, 10, 11, 12, 13, 14, 15]. On the other hand, certain amounts of SUSY can be broken by assigning proper boundary conditions to the high dimensional theory. For example, the 5D N=1 SUSY, which amounts to 4D

N=2 SUSY, can be broken to N=1 SUSY by orbifold projection. The possibility that the remaining N=1 SUSY is broken by boundary conditions is fairly attractive. Such SUSY breaking mechanism is elegant and also can be interpreted to have a dynamical origin through AdS/CFT correspondence [16] when such a theory is put in a Randall-Sundrum [17] type warped extra-dimension.

We know that the SUSY partners of the SM particles can acquire masses after SUSY breaking. Naturalness argument requires weak-scale soft SUSY parameters. However, current collider experiments severely constrained the parameter space of the MSSM, e.g., the LHC has pushed the first two generations of squarks above TeV-scale. Models of natural SUSY [18] seek to retain the naturalness of SUSY by positing a spectrum of light higgsinos and light top squarks along with very heavy masses of other squarks and TeV-scale gluinos¹. Such models have low electroweak fine-tuning and satisfy the LHC constraints. Besides, a relatively heavy (125 GeV) Higgs mass indicates that natural SUSY may take the form of radiative natural SUSY [20]. In this paper, we propose an SU(5) orbifold GUT model which can reduce to the low energy natural SUSY after integrating out the heavy modes. Like other GUT models, our scenario has only a few free parameters.

This paper is organized as follows. In Section 2 we briefly review the 4D SU(5) GUT and possible non-minimal Kahler potential extensions. In Section 3 we obtain the soft SUSY breaking terms from the boundary conditions. In Section 4 we propose an approach to obtain a large trilinear coupling for stop and discuss the relevant consistency conditions for the free parameters in our theory. In Section 5 we scan the parameter space under current experimental constraints. Section 6 contains our conclusions.

2. A brief review of Grand Unified Theories

In this section we briefly review the SU(5) GUT model and present our conventions. We denote the left-handed quark doublets, the right-handed up-type quarks, the right-handed down-type quarks, the left-handed lepton doublets, the right-handed neutrinos and the right-handed charged leptons as Q_i , U_i^c , D_i^c , L_i , N_i^c , and E_i^c , respectively. Besides, we denote the two Higgs doublets in minimal supersymmetric standard model as h_u and h_d , which are introduced to give masses to the up-type and down-type quarks, respectively.

In $SU(5)$ model, the $U(1)_Y$ hypercharge generator are defined to be

$$T_{U(1)_Y} = \text{diag} \left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2} \right) . \quad (2.1)$$

The $SU(5)$ representations are decomposed in terms of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry as

$$\mathbf{5} = (\mathbf{3}, \mathbf{1}, -1/3) \oplus (\mathbf{1}, \mathbf{2}, 1/2) , \quad (2.2)$$

$$\bar{\mathbf{5}} = (\bar{\mathbf{3}}, \mathbf{1}, 1/3) \oplus (\mathbf{1}, \mathbf{2}, -1/2) , \quad (2.3)$$

¹In the framework of MSSM a 125 GeV Higgs mass requires heavy stops or large A_t . In order to have weak-scale stops, the MSSM must be extended, e.g., by introducing a gauge singlet superfield (for a comparative study of low energy SUSY models in light of the LHC Higgs data, see [19]).

$$\mathbf{10} = (\mathbf{3}, \mathbf{2}, \mathbf{1/6}) \oplus (\bar{\mathbf{3}}, \mathbf{1}, -\mathbf{2/3}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}) , \quad (2.4)$$

$$\bar{\mathbf{10}} = (\bar{\mathbf{3}}, \mathbf{2}, -\mathbf{1/6}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{2/3}) \oplus (\mathbf{1}, \mathbf{1}, -\mathbf{1}) , \quad (2.5)$$

$$\mathbf{24} = (\mathbf{8}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{3}, \mathbf{2}, -\mathbf{5/6}) \oplus (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{5/6}) . \quad (2.6)$$

There are three families of the standard model matter contents with their quantum numbers under $SU(5)$

$$F'_i = \mathbf{10}, \quad \bar{f}'_i = \bar{\mathbf{5}}, \quad N_i^c = \mathbf{1} , \quad (2.7)$$

where $i = 1, 2, 3$ denote the three families. The standard model particle assignments in $SU(5)$ multiplets F'_i and \bar{f}'_i are

$$F'_i = (Q_i, U_i^c, E_i^c) , \quad \bar{f}'_i = (D_i^c, L_i) . \quad (2.8)$$

To break the $SU(5)$ GUT gauge symmetry and further electroweak gauge symmetry, we introduce an adjoint Higgs field and two fundamental representation Higgs fields whose quantum numbers under $SU(5)$ are

$$\Phi' = \mathbf{24} , \quad h' = \mathbf{5} , \quad \bar{h}' = \bar{\mathbf{5}} , \quad (2.9)$$

where h' and \bar{h}' contain the Higgs doublets h_u and h_d , respectively.

The supegravity scalar potential can be written as [21]

$$V = M_*^4 e^G \left[G^i (G^{-1})^j_i G_j - 3 \right] + \frac{1}{2} \text{Re} \left[(f^{-1})_{ab} \hat{D}^a \hat{D}^b \right] , \quad (2.10)$$

where M_* is the reduced Planck scale and the D-terms are given by

$$\hat{D}^a \equiv - G^i (T^a)^j_i \phi_j = - \phi^{j*} (T^a)^i_j G_i . \quad (2.11)$$

The Kähler function G as well as its derivatives and metric G_i^j are given by

$$G \equiv \frac{K}{M_*^2} + \ln \left(\frac{W}{M_*^3} \right) + \ln \left(\frac{W^*}{M_*^3} \right) , \quad (2.12)$$

$$G^i = \frac{\delta G}{\delta \phi_i} , \quad G_i = \frac{\delta G}{\delta \phi_i^*} , \quad G_i^j = \frac{\delta^2 G}{\delta \phi_i^* \delta \phi_j} , \quad (2.13)$$

where K is the Kähler potential and W is the superpotential.

We consider the following Kähler potential

$$K = a_0 \phi_i^\dagger e^{2gV} \phi_i + \frac{b_\Phi}{M_*} \phi_i^\dagger e^{2gV} \Phi \phi_i + \frac{b_S}{M_*} S \phi_i^\dagger e^{2gV} \phi_i . \quad (2.14)$$

and superpotential

$$W = \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k + \frac{1}{6} \alpha_\Phi^{ijk} \frac{\Phi}{M_*} \phi_i \phi_j \phi_k . \quad (2.15)$$

When the GUT gauge symmetry is broken by giving lowest component vevs to the Higgs chiral multiplets Φ and S , we have the general superpotential and Kähler potential [22, 23]

$$K = a_0 \phi_i^\dagger e^{2gV} \phi_i + b_\Phi \phi_i^\dagger e^{2gV} \frac{\langle \Phi \rangle}{M_*} \phi_i + b_S \frac{\langle S \rangle}{M_*} \phi_i^\dagger e^{2gV} \phi_i , \quad (2.16)$$

$$W = \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k + \frac{1}{6} \alpha_\Phi^{ijk} \frac{\langle \Phi \rangle}{M_*} \phi_i \phi_j \phi_k . \quad (2.17)$$

In general, the non-universal gaugino masses, the SUSY-breaking scalar masses and the trilinear soft terms will be obtained with such higher dimensional operators.

3. SUSY soft masses from Scherk-Schwarz mechanism

We consider the five-dimensional space-time $\mathcal{M}_4 \times S^1/Z_2$ comprising of Minkowski space \mathcal{M}_4 with coordinates x_μ and the orbifold S^1/Z_2 with coordinate $y \equiv x_5$. The orbifold S^1/Z_2 is obtained from S^1 by moduling the equivalent classes:

$$P : y \sim -y . \quad (3.1)$$

There are two inequivalent 3-branes locating at $y = 0$ and $y = \pi R$ which are denoted as O and O' , respectively.

It is well known that the five dimensional $N = 1$ supersymmetric gauge theory corresponds to $N = 2$ supersymmetry in four dimensions. The vector multiplet in $N=2$ SUSY contains a vector boson A_M with $M = 0, 1, 2, 3, 5$ the space-time coordinates, two Weyl gauginos $\lambda_{1,2}$, and a real scalar field σ . In terms of four-dimensional $N = 1$ supersymmetry, it contains a vector multiplet $V(A_\mu, \lambda_1)$ and a chiral multiplet $\Sigma((\sigma + iA_5)/\sqrt{2}, \lambda_2)$ which transform in the adjoint representation of the gauge group. The hypermultiplet in $N=2$ supersymmetry has two complex scalars ϕ and ϕ^c , a Dirac fermion Ψ . It can be decomposed (in terms $N=1$ SUSY fields) into two 4-dimensional chiral multiplets $\Phi(\phi, \psi \equiv \Psi_R)$ and $\Phi^c(\phi^c, \psi^c \equiv \Psi_L)$, which transform as conjugate representations of each other under the gauge group.

The action [24] for the vector multiplet and its couplings to the bulk hypermultiplet Φ are given by

$$\begin{aligned} S = & \int d^5x \frac{1}{kg^2} \text{Tr} \left[\frac{1}{4} \int d^2\theta (W^\alpha W_\alpha + \text{H.C.}) \right. \\ & + \int d^4\theta \left((\sqrt{2}\partial_5 + \bar{\Sigma})e^{-V} (-\sqrt{2}\partial_5 + \Sigma)e^V + \partial_5 e^{-V} \partial_5 e^V \right) \Big] \\ & + \int d^5x \left[\int d^4\theta (\Phi^c e^V \bar{\Phi}^c + \bar{\Phi} e^{-V} \Phi) \right. \\ & \left. + \int d^2\theta \left(\Phi^c (\partial_5 - \frac{1}{\sqrt{2}}\Sigma) \Phi + \text{H.C.} \right) \right] . \end{aligned} \quad (3.2)$$

To ensure the action to be invariant under the parity P operation, we can obtain the transformation rule for vector multiplet

$$V(x^\mu, y) \rightarrow V(x^\mu, -y) = PV(x^\mu, y)P^{-1} , \quad (3.3)$$

$$\Sigma(x^\mu, y) \rightarrow \Sigma(x^\mu, -y) = -P\Sigma(x^\mu, y)P^{-1} . \quad (3.4)$$

When the hypermultiplet belongs to the fundamental or anti-fundamental representation, invariance of the action requires

$$\Phi(x^\mu, y) \rightarrow \Phi(x^\mu, -y) = \eta_\Phi P \Phi(x^\mu, y) , \quad (3.5)$$

$$\Phi^c(x^\mu, y) \rightarrow \Phi^c(x^\mu, -y) = -\eta_\Phi P \Phi^c(x^\mu, y) . \quad (3.6)$$

Similarly, if the hypermultiplet belongs to the symmetric, anti-symmetric or adjoint representation, the transformation rules are

$$\Phi(x^\mu, y) \rightarrow \Phi(x^\mu, -y) = \eta_\Phi P \Phi(x^\mu, y) P , \quad (3.7)$$

$$\Phi^c(x^\mu, y) \rightarrow \Phi^c(x^\mu, -y) = -\eta_\Phi P \Phi^c(x^\mu, y) P . \quad (3.8)$$

where $\eta_\Phi = \pm 1$.

The gauge symmetry and supersymmetry can be broken by suitable boundary conditions with proper chosen representations for parity P . We denote the transformation of a field ϕ in the representation of unbroken gauge symmetry as

$$\phi(x_\mu, y) \rightarrow \phi(x_\mu, -y) = p_\phi \phi(x_\mu, y) , \quad (3.9)$$

where $p_\phi = \pm 1$.

The non-trivial twist T for translation

$$T\phi(x_\mu, y) = \phi(x_\mu, y + 2\pi R) , \quad (3.10)$$

should satisfy the following consistency condition between the orbifolding P'' and the translation

$$TP''T = P'' . \quad (3.11)$$

Thus it must have the form [25, 26]

$$T = \exp(-2\pi i \sigma_2 \alpha) \quad (3.12)$$

for non-trivial orbifolding projection of $SU(2)_R$ global symmetry

$$P'' = \sigma_3 . \quad (3.13)$$

We can simultaneously impose the gauge symmetry breaking as well as supersymmetry breaking boundary conditions in our scenario. We impose trivial orbifold projection conditions and non-trivial twisting to break the gauge symmetry. The nontrivial twisting

$$V(x_\mu, y + 2\pi R) = TV(x_\mu, y)T^{-1} , \quad (3.14)$$

$$H(x_\mu, y + 2\pi R) = TH(x_\mu, y) \quad (3.15)$$

can break the gauge symmetry in certain fixed points of the orbifold. An alternative way to see the gauge symmetry breaking by boundary conditions is that the translation twist

T and reflection P_0 at $y = 0$ can be combined to give the reflection at $y = \pi R$ with the reflection operator

$$P_{\pi R} \equiv T P_0 : y + \pi R \rightarrow -y + \pi R. \quad (3.16)$$

Then the massless zero modes can preserve different gauge symmetries by assigning proper $(P_0, P_{\pi R})$ boundary conditions to the two fix points. Only $\phi_{++}(x_\mu, y)$ possess a four-dimensional massless zero mode. It is easy to see that ϕ_{++} and ϕ_{+-} are non-vanishing at $y = 0$ brane, and ϕ_{++} and ϕ_{-+} are non-vanishing at $y = \pi R$ brane.

We choose

$$P_0 = (+1, +1, +1, +1, +1), \quad T = (+1, +1, +1, -1, -1), \quad (3.17)$$

so that the boundary conditions preserve the $SU(5)$ gauge symmetry on the $y = 0$ brane as well as in the bulk while break the $SU(5)$ gauge symmetry down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ on the $y = \pi R$ brane. In this scenario with $1/R \sim 10^{16}$ GeV, the undesirable doublet-triplet splitting problem is solved by orbifold projection. Besides, the boundary conditions will break $N = 2$ SUSY to $N = 1$ SUSY.

The remaining $N = 1$ SUSY breaking can be realized via the Scherk-Schwarz mechanism through the boundary conditions [27, 28]. It is well known that $N = 1$ SUSY in five dimensions possesses an $SU(2)_R$ global R -symmetry under which the gauginos from the vector multiplets (λ_1, λ_2) and complex scalars (ϕ, ϕ^\dagger) from hypermultiplets form $SU(2)_R$ doublets. In SUSY $SU(5)$ GUT, we need two hypermultiplets H_1 and H_2 in **5** representation of $SU(5)$ GUT group to give low energy Higgs fields h_u, h_d after orbifolding. So the boundary conditions adopted in our scenario can be written as

$$\begin{aligned} \begin{pmatrix} V \\ \Sigma \end{pmatrix} (x_\mu, -y) &= \begin{pmatrix} V \\ -\Sigma \end{pmatrix} (x_\mu, y), \\ \begin{pmatrix} \Phi \\ \Phi^c \end{pmatrix} (x_\mu, -y) &= \begin{pmatrix} \Phi \\ -\Phi^c \end{pmatrix} (x_\mu, y). \end{aligned} \quad (3.18)$$

with $\eta_{\phi_1} = -\eta_{\phi_2}$ and

$$A^M(x^\mu, y + 2\pi R) = T A^M(x^\mu, y) T, \quad (3.19)$$

$$\sigma(x^\mu, y + 2\pi R) = T \sigma(x^\mu, y) T, \quad (3.20)$$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} (x^\mu, y + 2\pi R) = e^{-2\pi i \alpha \sigma_2} T \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} (x^\mu, y), \quad (3.21)$$

$$\begin{pmatrix} \phi_1 & \phi_2 \\ \phi_1^{c\dagger} & \phi_2^{c\dagger} \end{pmatrix} (x^\mu, y + 2\pi R) = e^{-2\pi i \alpha \sigma_2} T \begin{pmatrix} \phi_1 & \phi_2 \\ \phi_1^{c\dagger} & \phi_2^{c\dagger} \end{pmatrix} (x^\mu, y), \quad (3.22)$$

$$\begin{pmatrix} \tilde{\phi}_1 & \tilde{\phi}_2 \\ \tilde{\phi}_1^{c\dagger} & \tilde{\phi}_2^{c\dagger} \end{pmatrix} (x^\mu, y + 2\pi R) = T \begin{pmatrix} \tilde{\phi}_1 & \tilde{\phi}_2 \\ \tilde{\phi}_1^{c\dagger} & \tilde{\phi}_2^{c\dagger} \end{pmatrix} (x^\mu, y). \quad (3.23)$$

The $SU(2)_R$ R -symmetry satisfies the consistency conditions because the orbifold parity is proportional to σ_3 . For trivial boundary conditions of reflection at $y = 0$ with respect to

gauge symmetry, the consistency conditions only require $T^2 = 1$ which is trivially satisfied in our scenario. Similar to the Higgs cases, squarks and sleptons of the first two generations can have non-trivial boundary conditions. In order to give low energy matter spectrum of MSSM, we have to introduce two hypermultiplets $10^A, 10^B$ ($\bar{5}^A, \bar{5}^B$) for each type of representations in $SU(5)$. The decomposition of the matter content with respect to the orbifolding in terms of the $(P_0, P_{\pi R})$ parity assignment is

$$10_i^A = (\bar{\mathbf{3}}, \mathbf{1})_{-4/3}^{(+,+)} \oplus (\mathbf{3}, \mathbf{2})_{1/3}^{(+,-)} \oplus (\mathbf{1}, \mathbf{1})_2^{(+,+)}, \quad (3.24)$$

$$10_i^B = (\bar{\mathbf{3}}, \mathbf{1})_{-4/3}^{(+,-)} \oplus (\mathbf{3}, \mathbf{2})_{1/3}^{(+,+)} \oplus (\mathbf{1}, \mathbf{1})_2^{(+,-)}, \quad (3.25)$$

$$\bar{5}_i^A = (\bar{\mathbf{3}}, \mathbf{1})_{2/3}^{(+,+)} \oplus (\mathbf{1}, \mathbf{2})_{-1}^{(+,-)}, \quad (3.26)$$

$$\bar{5}_i^B = (\bar{\mathbf{3}}, \mathbf{1})_{2/3}^{(+,-)} \oplus (\mathbf{1}, \mathbf{2})_{-1}^{(+,+)}, \quad (3.27)$$

with $i = 1, 2$ being the family index. In our scenario, we place the first two generation matter contents on the bulk while the third generation matter contents on the $y = 0$ brane. This scenario will lead to natural SUSY in the IR limit after we integrate out the heavy modes.

We find that it is non-trivial to adopt other global symmetries in addition to the R -symmetry $SU(2)_R$ in our scenario because of the Yukawa couplings. In order to accommodate both the global symmetry for the Higgs sector and the Yukawa couplings between the first two generation matter contents and the Higgs field, we must introduce additional Higgs hypermultiplet field ϕ_3 to ensure the Higgs sector transforms in $\mathbf{3}$ representation of the $SU(2)_V$ symmetry. Besides, global symmetry in the Yukawa sector also requires that we must assign the following transformation law (fundamental representation for $SU(2)_V$) for matter contents

$$\begin{aligned} \begin{pmatrix} 10_i^A & 10_i^B \\ \overline{10}_i^A & \overline{10}_i^B \end{pmatrix} (x^\mu, y + 2\pi R) &= \begin{pmatrix} 10_i^A & 10_i^B \\ \overline{10}_i^A & \overline{10}_i^B \end{pmatrix} (x^\mu, y) e^{2\pi i \gamma \sigma_2}, \\ \begin{pmatrix} \bar{5}_i^A & \bar{5}_i^B \\ \bar{5}_i^A & \bar{5}_i^B \end{pmatrix} (x^\mu, y + 2\pi R) &= \begin{pmatrix} \bar{5}_i^A & \bar{5}_i^B \\ \bar{5}_i^A & \bar{5}_i^B \end{pmatrix} (x^\mu, y) e^{2\pi i \gamma \sigma_2}, \end{aligned} \quad (3.28)$$

with the subscript $i = 1, 2$ for each generation.

The $\mathbf{3}$ representation of $SU(2)_V$ can be generated by an $SO(3)$ generator

$$T' = \exp \left(2\pi i \sum_{a=1}^3 T^a \theta^a \right), \quad (3.29)$$

with

$$T^1 = \begin{pmatrix} & -i \\ i & \\ & 0 \end{pmatrix}, \quad T^2 = \begin{pmatrix} & i \\ & 0 \\ -i & \end{pmatrix}, \quad T^3 = \begin{pmatrix} 0 & \\ & -i \\ & i \end{pmatrix}. \quad (3.30)$$

The consistency condition $T' P' T' = P'$ requires

$$\{P', \sum_a T_a \theta^a\} = 0. \quad (3.31)$$

We require ϕ_i adopt the following non-trivial boundary conditions

$$\begin{aligned} P_0(\phi_1) &= (+1, +1, +1, +1, +1) , & P_{\pi R}(\phi_1) &= (-1, -1, -1, +1, +1), \\ P_0(\phi_2) &= (+1, +1, +1, +1, +1) , & P_{\pi R}(\phi_2) &= (+1, +1, +1, -1, -1), \\ P_0(\phi_3) &= (-1, -1, -1, -1, -1) , & P_{\pi R}(\phi_3) &= (+1, +1, +1, -1, -1). \end{aligned} \quad (3.32)$$

Then the most general flavor rotation that can be compatible with the projection

$$P'_0 = (+1, +1, -1) , \quad P'_{\pi R} = (-1, +1, +1), \quad (3.33)$$

are given by

$$T' = \exp \left(2\pi i [T^2 \theta^2 + T^3 \theta^3] \right). \quad (3.34)$$

The boundary conditions for the Higgs sector can be written as

$$\begin{aligned} \begin{pmatrix} \phi_1 & \phi_2 & \phi_3 \\ \phi_1^{c\dagger} & \phi_2^{c\dagger} & \phi_3^{c\dagger} \end{pmatrix} (x^\mu, y + 2\pi R) &= e^{-2\pi i \alpha \sigma_2} T \begin{pmatrix} \phi_1 & \phi_2 & \phi_3 \\ \phi_1^{c\dagger} & \phi_2^{c\dagger} & \phi_3^{c\dagger} \end{pmatrix} (x^\mu, y) e^{2\pi i [\theta^2 T^2 + \theta^3 T^3]}, \\ \begin{pmatrix} \tilde{\phi}_1 & \tilde{\phi}_2 & \tilde{\phi}_3 \\ \tilde{\phi}_1^{c\dagger} & \tilde{\phi}_2^{c\dagger} & \tilde{\phi}_3^{c\dagger} \end{pmatrix} (x^\mu, y + 2\pi R) &= T \begin{pmatrix} \tilde{\phi}_1 & \tilde{\phi}_2 & \tilde{\phi}_3 \\ \tilde{\phi}_1^{c\dagger} & \tilde{\phi}_2^{c\dagger} & \tilde{\phi}_3^{c\dagger} \end{pmatrix} (x^\mu, y) e^{2\pi i [\theta^2 T^2 + \theta^3 T^3]}. \end{aligned} \quad (3.35)$$

The relation between the parameter θ and γ is determined by the relation

$$U \sigma^a U^{-1} = \sum_a \sigma^b R_{ab}. \quad (3.36)$$

Using the expression

$$U = e^{2\pi i \gamma \sigma_2} \quad (3.37)$$

and the commutation relation for σ_a , we can obtain the relation between γ and θ^i

$$\theta^2 = 2\gamma , \quad \theta^3 = 0. \quad (3.38)$$

So the transformation law for the Higgs fields, which is compatible with the global symmetry, can be written as

$$R = e^{4\pi i \gamma T^2}. \quad (3.39)$$

It is well known that CP-violation constraints require heavy superpartners for light quarks. In order to assign such heavy sparticle masses to the first two generations, we require that there is an additional flavor symmetry $SU(2)_{V'}$ for the first two generations

$$\begin{aligned} \begin{pmatrix} 10_i^A & 10_i^B \\ \overline{10}_i^A & \overline{10}_i^B \end{pmatrix} (x^\mu, y + 2\pi R) &= \begin{pmatrix} 10_i^A & 10_i^B \\ \overline{10}_i^A & \overline{10}_i^B \end{pmatrix} (x^\mu, y) e^{2\pi i \beta \sigma_2}, \\ \begin{pmatrix} \bar{5}_i^A & \bar{5}_i^B \\ 5_i^A & 5_i^B \end{pmatrix} (x^\mu, y + 2\pi R) &= \begin{pmatrix} \bar{5}_i^A & \bar{5}_i^B \\ 5_i^A & 5_i^B \end{pmatrix} (x^\mu, y) e^{2\pi i \beta \sigma_2}, \end{aligned} \quad (3.40)$$

with β being the parameter for $SU(2)_{V'}$ flavor symmetry. The Higgs fields are singlet with respect to this flavor symmetry and thus they receive no contributions from this global symmetry twisting. The expansion of zero modes can then be written as

$$\begin{pmatrix} \lambda_1^a(++) \\ \lambda_2^a(--) \end{pmatrix} (x^\mu, y) = \sum_n e^{-i\alpha\sigma_2 y/R} \begin{pmatrix} \lambda_{1n}^a \cos \left[\frac{ny}{R} \right] \\ \lambda_{2n}^a \sin(n+1)\frac{y}{R} \end{pmatrix}, \quad (3.41)$$

for gauginos. Similarly for the Higgs sector we have

$$\begin{aligned} \begin{pmatrix} \phi_1 & \phi_2 & \phi_3 \\ \phi_1^{c\dagger} & \phi_2^{c\dagger} & \phi_3^{c\dagger} \end{pmatrix} (x^\mu, y) &= e^{-i\frac{\alpha}{R}\sigma_2 y} \begin{pmatrix} \phi_1[+, \pm] & \phi_2[+, \mp] & \phi_3[-, \mp] \\ \phi_1^{c\dagger}[-, \mp] & \phi_2^{c\dagger}[-, \pm] & \phi_3^{c\dagger}[+, \pm] \end{pmatrix} (x^\mu, y) e^{i2\frac{\gamma y}{R}T^2}, \\ \begin{pmatrix} \tilde{\phi}_1 & \tilde{\phi}_2 & \tilde{\phi}_3 \\ \tilde{\phi}_1^{c\dagger} & \tilde{\phi}_2^{c\dagger} & \tilde{\phi}_3^{c\dagger} \end{pmatrix} (x^\mu, y) &= \begin{pmatrix} \tilde{\phi}_1[+, \pm] & \tilde{\phi}_2[+, \mp] & \tilde{\phi}_3[-, \mp] \\ \tilde{\phi}_1^{c\dagger}[-, \mp] & \tilde{\phi}_2^{c\dagger}[-, \pm] & \tilde{\phi}_3^{c\dagger}[+, \pm] \end{pmatrix} (x^\mu, y) e^{i2\frac{\gamma y}{R}T^2}, \end{aligned} \quad (3.42)$$

with the decomposition depending on the orbifold projection

$$\begin{aligned} \psi[+, +](x^\mu, y) &= \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n,0}} \pi R}} \psi_{+,+}^n(x^\mu) \cos n \frac{y}{R}, \\ \psi[+, -](x^\mu, y) &= \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \psi_{+,-}^n(x^\mu) \cos(n + \frac{1}{2}) \frac{y}{R}, \\ \psi[-, +](x^\mu, y) &= \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \psi_{-,+}^n(x^\mu) \sin(n + \frac{1}{2}) \frac{y}{R}, \\ \psi[-, -](x^\mu, y) &= \sum_{n=0}^{\infty} \frac{1}{\sqrt{\pi R}} \psi_{-,-}^n(x^\mu) \sin(n+1) \frac{y}{R}. \end{aligned} \quad (3.43)$$

After we integrate out the heavy Kaluza-Klein modes, we can obtain the contributions of the SUSY soft parameters from the twisting boundary conditions

$$\begin{aligned} \Delta \mathcal{L} &\supseteq -\frac{1}{2} \frac{\alpha}{R} \left(\tilde{G}^a \tilde{G}^a + \tilde{W}^a \tilde{W}^a + \tilde{B} \tilde{B} \right) - \left(\frac{\alpha^2}{R^2} + 4 \frac{\gamma^2}{R^2} \right) (h_u^2 + h_d^2) \\ &+ \frac{4\alpha\gamma}{R^2} h_u h_d - \frac{2\gamma}{R} \tilde{h}_u \tilde{h}_d - \left(\frac{\alpha^2}{R^2} + \frac{\beta^2}{R^2} + \frac{\gamma^2}{R^2} \right) \sum_{i=1}^2 \left(|\tilde{Q}_L^i|^2 + |\tilde{U}_R^i|^2 + |\tilde{D}_R^i|^2 \right) \\ &- \left(\frac{\alpha^2}{R^2} + \frac{\beta^2}{R^2} + \frac{\gamma^2}{R^2} \right) \sum_{i=1}^2 \left(|\tilde{L}_L^i|^2 + |\tilde{E}_R^i|^2 \right), \end{aligned} \quad (3.44)$$

with $i = 1, 2$ being the family index. In order to have the soft parameters at TeV scale, we require $\alpha, \gamma \ll 1$, $\alpha/R \sim \mathcal{O}(\text{TeV})$ and $\gamma/R \sim \mathcal{O}(100 \text{ GeV})$. An $SU(3)_c$ triplet Higgs field survives the orbifold projection. Flavor symmetry guarantees that this triplet is inert. For simply, we can add heavy brane mass terms for the $SU(3)_c$ triplet and integrate out this field so that they do not appear in the low energy spectrum. In fact, there is an alternative possibility concerning the boundary condition of ϕ_2 . We can choose the boundary conditions for ϕ_2 so that another $SU(2)_L$ doublet will survive orbifold projection. We will leave this possibility in our subsequent works.

4. Natural SUSY with a large A_t term

In natural SUSY the first/second generation squarks and sleptons are very heavy so that they are beyond the LHC reach and also possibly heavy enough to provide a (partial) decoupling solution to the SUSY flavor and CP problems. Naturalness requires that the third generation sfermions are not too heavy. In order to have light third generations in our scenario, we can put the third generation matter contents in the $y = 0$ brane. Thus there are no boundary breaking contributions to the sfermions of the third generation. The third generation squark masses can be generated by gaugino loops:

$$m_{\tilde{f}}^2 = \sum_i \frac{g_i^2}{16\pi^2} M_{\lambda_i}^2 = \sum_I \frac{g_U^2}{16\pi^2} (M_\lambda^U)^2, \quad (4.1)$$

where 'I' denotes the number of interaction types involved in the loops. This predicts that the stop masses are heavier than other sfermions of the third generation. At the same time, the contributions of gaugino loops to the first two generations are subleading. The soft masses for the third generation at the compactification scale are

$$\begin{aligned} m_{\tilde{Q}_L^3}^2 &= \frac{121}{60} \frac{g_U^2}{16\pi^2} \frac{\alpha^2}{R^2}, \quad m_{\tilde{t}_L^c}^2 = \frac{19}{15} \frac{g_U^2}{16\pi^2} \frac{\alpha^2}{R^2}, \quad m_{\tilde{b}_L^c}^2 = \frac{16}{15} \frac{g_U^2}{16\pi^2} \frac{\alpha^2}{R^2}, \\ m_{\tilde{L}_L^3}^2 &= \frac{23}{20} \frac{g_U^2}{16\pi^2} \frac{\alpha^2}{R^2}, \quad m_{\tilde{\tau}_L^c}^2 = \frac{3}{5} \frac{g_U^2}{16\pi^2} \frac{\alpha^2}{R^2}, \quad m_{\tilde{\nu}_{\tau_L^c}}^2 = 0. \end{aligned} \quad (4.2)$$

While the advantages of natural SUSY are obvious (low EWFT, decoupling solution to SUSY flavor and CP problems), some apparent problems seem to arise. First, the sub-TeV top squarks usually lead to m_h in the 115-120 GeV range, below 125 GeV. The approximate one-loop formula for the Higgs mass [29] is given by

$$m_h^2 = m_Z^2 \cos^2 2\beta + \frac{3m_t^4}{4\pi^2 v^2} \left[\ln \left(\frac{M_S^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left(1 - \frac{X_t^2}{12M_S^2} \right) \right], \quad (4.3)$$

with

$$X_t \equiv A_t - \mu \cot \beta, \quad M_S^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}. \quad (4.4)$$

An interesting observation is that the Higgs mass, as a function of X_t/M_S , is maximal with the maximal mixing scenario $X_t/M_S = \sqrt{6}$ [30]. So a large trilinear term A_t can help to push up the Higgs mass.

In our scenario the trilinear terms can be generated by SUSY breaking boundary conditions. The Lagrangian on the $y = 0$ brane gives

$$\mathcal{L} \supseteq \int d^2\theta \delta(y) [y_2 F'_3 \bar{f}_3 \phi_3^c + y_3 F'_3 F'_3 \phi_1], \quad (4.5)$$

which leads to

$$A_t = -y_3 \frac{\alpha}{R} \sim -\frac{\alpha}{R} \quad (4.6)$$

after the F-term of the bulk hypermultiplet fields ϕ_1 , which is $F_{\phi_1} = -(\partial_{\phi_1} W)^* - \partial_y \phi_1^c$, is substituted into the superpotential. We should note that the trilinear term A_t is independent of γ even when such an additional flavor symmetry is present. At the compactification scale, the ratio

$$\frac{X_t}{M_S} \approx \frac{A_t}{\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}} \sim -4\pi. \quad (4.7)$$

is fairly large. The absolute value of A_t tends to increase with Renormalization Group Equation running to lower scale which would give an even larger X_t/M_S . So our scenario naturally gives a large A_t to explain the observed 125 GeV Higgs. On the other hand, the maximal mixing requires a not too large A_t value. So we will give a moderately large A_t by introducing non-minimal Kahler potential.

In our setting, the following interactions are located in the $y=0$ brane which is $SU(5)$ preserving. We introduce the general Kähler potential for the third generation matter content

$$K = \phi^\dagger e^{2gV} \phi + b_S \frac{S}{M_*} \phi^\dagger e^{2gV} \phi, \quad (4.8)$$

with S being a gauge singlet chiral field. After the singlet S develops a vacuum expectation value (vev), we get

$$K = (1 + b_S \frac{\langle S \rangle}{M_*}) \phi^\dagger e^{2gV} \phi. \quad (4.9)$$

Because S is a gauge singlet, it can acquire a vev of order M_* .

We will introduce non-renormalizable Kähler terms for the two Higgs fields in $\mathbf{5}, \bar{\mathbf{5}}$ representations of $SU(5)$. Then we can obtain the wave function normalization for h_u and h_d

$$Z_{h_u} = 1 + b_S^u \frac{\langle S \rangle}{M_*}, \quad Z_{h_d} = 1 + b_S^d \frac{\langle S \rangle}{M_*}. \quad (4.10)$$

Thus the mass terms appeared in previous section contributed from boundary conditions are rescaled as

$$\begin{aligned} m_{h_u}^2 &= \frac{1}{Z_{h_u} R^2} (\alpha^2 + 4\gamma^2), \\ m_{h_d}^2 &= \frac{1}{Z_{h_d} R^2} (\alpha^2 + 4\gamma^2), \end{aligned} \quad (4.11)$$

as well as the terms for μ and $B\mu$

$$\begin{aligned} \mu &= \frac{2\gamma}{\sqrt{Z_{h_u} Z_{h_d}} R}, \\ B\mu &= \frac{-4\alpha\gamma}{\sqrt{Z_{h_u} Z_{h_d}} R^2}. \end{aligned} \quad (4.12)$$

We adopt the choice that $m_{h_u}^2 < m_{h_d}^2$ with $Z_{h_u} > Z_{h_d}$. Such rescaling changes the UV input to $m_{h_u}^2 \neq m_{h_d}^2$ to avoid possible problems related to radiative electroweak gauge symmetry breaking which appears in mSUGRA and GMSB.

With the previous non-minimal kinetic mass terms for h_u ², the trilinear coupling can be rescaled as

$$A'_t = \frac{A_t}{\sqrt{Z_{h_u}}} = -y_t \frac{\alpha}{\sqrt{Z_{h_u}} R}. \quad (4.13)$$

Thus the trilinear coupling could be rescaled to a moderately large value in our scenario. Same arguments give $A'_b = -y_b \frac{\alpha}{\sqrt{Z_{h_d}} R}$.

Constraints from the LHC Higgs mass measurement suggest that the lower bound on $\tan \beta$ is $\tan \beta \gtrsim 3.5$ [31]. For such a large $\tan \beta$, successful electroweak symmetry breaking gives the tree-level relation

$$\frac{m_Z^2}{2} = \frac{\tan^2 \beta m_{h_u}^2 - m_{h_d}^2}{1 - \tan^2 \beta} - |\mu|^2 \approx m_{h_u}^2 - |\mu|^2. \quad (4.14)$$

Naturalness condition requires $m_{h_u}^2(M_{SUSY}) \sim \mu^2(M_{SUSY}) \sim M_Z^2/2$. This requirement can be estimated to be

$$\frac{\alpha}{\gamma} \approx \frac{Z_{h_u}}{Z_{h_d}}, \quad (4.15)$$

which can be relaxed if loop corrections are taken into account. The electroweak symmetry breaking also requires

$$|B\mu|^2 > (|\mu|^2 + m_{h_u}^2)(|\mu|^2 + m_{h_d}^2). \quad (4.16)$$

In our scenario, it can be written as

$$\frac{4\alpha^2\gamma^2}{Z_{h_u}Z_{h_d}R^4} > \frac{\alpha^2(\alpha^2 + 4\gamma^2)}{Z_{h_u}^2Z_{h_d}^2R^4}, \quad (4.17)$$

which can be simplified as

$$4Z_{h_u}Z_{h_d} > \frac{\alpha^2 + 4\gamma^2}{\gamma^2}. \quad (4.18)$$

In summary, our scenario has the following free parameters:

- The parameters α , β (with $\beta \gg \alpha$ to evade flavor constraints) and γ related to SUSY breaking boundary conditions so that $\alpha/R \sim \mathcal{O}(\text{TeV})$. Besides, natural SUSY requires a light Higgsino, which amounts to $\gamma/R \sim \mathcal{O}(\text{TeV})$ value after rescaling.
- The Higgs field wavefunction normalization Z_{h_u} and Z_{h_d} . Naturalness condition requires $Z_{h_u} > Z_{h_d} > 1$.

Then the SUSY parameters at the GUT scale are related to the above free parameters as

- The gaugino masses: $m_{1/2} = \frac{\alpha}{R}$.

²There are also possible brane localized kinetic terms for the third generation. Normalizing the kinetic term can also contribute an additional factor to the trilinear coupling.

- The sfermion masses for the first two generations given by $m_0^2 = \frac{1}{R^2}(\alpha^2 + \beta^2)$.
- The Higgs soft mass terms: $m_{h_u}^2 = \frac{\alpha^2 + 4\gamma^2}{Z_{h_u} R^2}$, $m_{h_d}^2 = \frac{\alpha^2 + 4\gamma^2}{Z_{h_d} R^2}$.
- The $\mu - B\mu$ term: $\mu = \frac{2\gamma}{\sqrt{Z_{h_u} Z_{h_d}} R}$, $B\mu = \frac{-4\alpha\gamma}{\sqrt{Z_{h_u} Z_{h_d}} R^2}$.
- The trilinear soft terms for the third generation: $A_t = -y_t \frac{\alpha}{\sqrt{Z_{h_u}} R}$, $A_b = -y_b \frac{\alpha}{\sqrt{Z_{h_d}} R}$.
- The sfermion masses for the third generation shown in Eq.(4.2).

5. Viable parameter space

The above soft supersymmetric parameters are obtained at the compactification scale. Low energy soft SUSY spectrum can be obtained by solving the Renormalization Group Equation at the weak scale. This procedure is done with the code **SuSpect 2.4.1** [32]. The unification of the matter contents eliminates many free parameters. With the remained free parameters, this scenario is greatly constrained by various experiments. In our study we consider the following experimental constraints on the parameter space:

- (1) The $b \rightarrow s\gamma$ decay branching ratio $\text{BF}(b \rightarrow s\gamma) = (3.55 \pm 0.26) \times 10^{-4}$ [33]. We require the theoretical value to be in the 3σ range of the experimental data.
- (2) The $B_s \rightarrow \mu^+ \mu^-$ from LHCb measurement [34]: $\text{BF}(B_s \rightarrow \mu^+ \mu^-) = 3.2_{-1.2}^{+1.5} \times 10^{-9}$. We require the theoretical value to be in the 3σ range.
- (3) The dark matter relic density $\Omega h^2 = 0.1126 \pm 0.0036$ from WMAP [35] and $\Omega h^2 = 0.1199 \pm 0.0027$ from Planck [36]. We require the relic density of the neutralino dark matter satisfy the 2σ upper bound.
- (4) The XENON100(2012) constraints on the dark matter scattering off the nucleon [37].

Note that we do not impose $(g-2)_\mu$ constraints [38] in our scenario. Due to heavy $\tilde{\mu}_{1,2}$ and $\tilde{\nu}_\mu$, the SUSY contribution to $(g-2)_\mu$ is small in our scenario.

Under above constraints, we scan over the parameter space in our scenario with the code **MicrOmega 1.3** [39] and **SUSY_FLAVOR v2.02** [40]. In general, the parameters related to the electroweak symmetry breaking sector contain $m_{h_u}^2$, $m_{h_d}^2$, $|\mu|^2$, B_μ and $\tan\beta$. Successful radiative electroweak symmetry breaking requires these parameters to satisfy (4.14) and

$$\sin 2\beta = -\frac{2B_\mu}{m_{h_u}^2 + m_{h_d}^2 + 2|\mu|^2}. \quad (5.1)$$

Thus there remained essentially three parameters, which are taken as $\tan\beta$, μ and m_A at weak-scale. These parameters are related to $m_{h_u}^2$, $m_{h_d}^2$, $|\mu|^2$ and B_μ by

$$m_A^2 = \frac{2B_\mu}{\sin 2\beta} = 2|\mu|^2 + m_{h_u}^2 + m_{h_d}^2. \quad (5.2)$$

Also, these three free parameters are related to Z_{h_u} , Z_{h_d} and γ by Eqs.(4.11) and (4.12). In natural SUSY, the soft masses of the first two generations are of order $\mathcal{O}(10)$ TeV. So we can choose $\beta/R = 10$ TeV. We search for natural SUSY solutions in the parameter space in our scenario and perform a random scan in the following ranges:

$$\begin{aligned}
100 \text{ GeV} &< m_{1/2} < 4 \text{ TeV}, \\
100 \text{ GeV} &< -\mu < 150 \text{ GeV}, \\
150 \text{ GeV} &< m_A < 1.5 \text{ TeV}, \\
3 &< \tan \beta < 50, \\
-4 &< A_t/m_{1/2} < 4.
\end{aligned} \tag{5.3}$$

In our scan, we choose the top quark mass to be 175 GeV. The $SU(2)_L$ and $U(1)_Y$ gauge couplings at the weak scale lead to the following GUT-scale boundary conditions

$$\alpha_{GUT} = \frac{1}{24.3}, \quad M_{GUT} = 2.0 \times 10^{16} \text{ GeV}, \quad y_t(M_{GUT}) = 0.51, \quad y_b(M_{GUT}) = 0.054. \tag{5.4}$$

The unification scale is determined by $g_2(M_{GUT}) = g_1(M_{GUT}) = g_{GUT}$. Note that due to the relatively large uncertainty of $SU(3)_c$ gauge coupling at the electroweak scale, the value of α_s at the electroweak scale is usually regarded as a GUT prediction by the RGE running of α_{GUT} .

Natural SUSY requires light stops. So we require $m_{\tilde{t}_1} < 1.5$ TeV and $m_{\tilde{t}_2} < 2$ TeV. In our previous discussion, the stop masses are generated via gaugino loops and satisfy the relation $M_{Q_L}^2 \approx 2 \frac{g_U^2}{16\pi^2} \frac{\alpha^2}{R^2}$ and $M_{t_L}^2 = \frac{19}{15} \frac{g_U^2}{16\pi^2} \frac{\alpha^2}{R^2}$. Thus, heavy gauginos will in general lead to heavy stop masses. So from stop masses we anticipate an upper limit for the universal $m_{1/2}$.

In Fig.1 we present the scatter plots for the survived samples. The samples which satisfy the constraints (1-4) but do not satisfy $m_{\tilde{t}_1} < 1.5$ TeV or $m_{\tilde{t}_2} < 2$ TeV are also displayed. We can see that the allowed $m_{1/2}$ value is below 1.5 TeV, which is mainly from the requirement of light stops. On the other hand, since low stop masses are difficult to push up the Higgs boson mass, only a few survived samples can give a SM-like Higgs boson in the range of 123-127 GeV. However, if we do not impose the stop mass upper limits, a 125 GeV SM-like Higgs can be obtained easily [41], as shown by this figure. This figure also shows the allowed ranges for other parameters. We can see that a negative A_t can not be too large. Besides, a large M_A and a moderate $\tan \beta$ is favored due to the flavor physics constraints.

We show in Fig.2 the neutralino relic density and the spin-independent neutralino-proton scattering cross section versus the neutralino mass. Due to the low value of μ , the neutralino χ_1^0 is higgsino-like which has large annihilation cross section and thus low relic density³. We also see from fig.2 that the samples with light stops give a larger relic density than the samples with heavy stops. The reason is that a small stop mass means a smaller

³In natural SUSY the neutralino dark matter is higgsino-like and its relic density is not sufficient to explain the measured value. This means that dark matter cannot be solely made of the neutralino and other components like axion should exist.

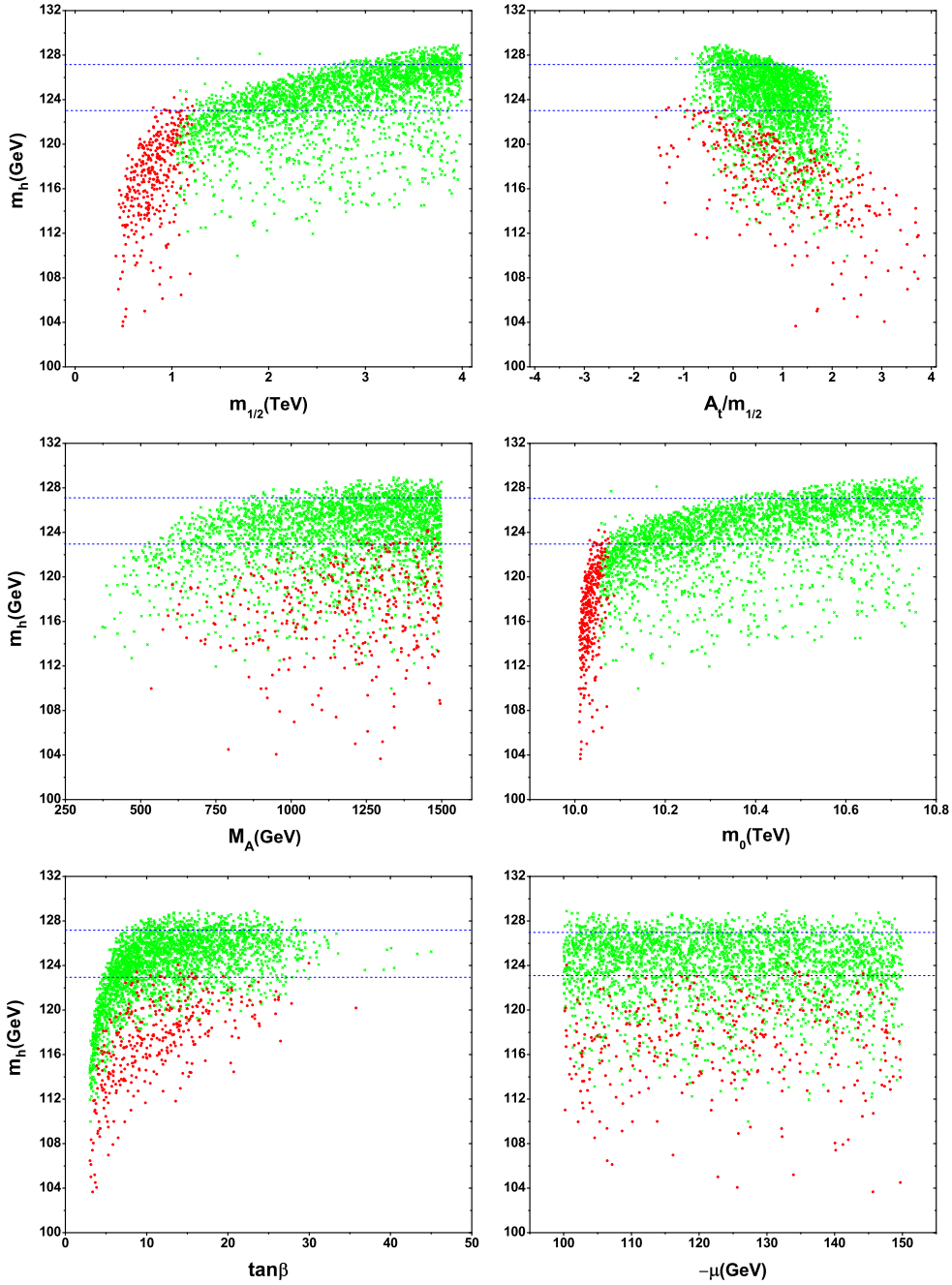


Figure 1: Scatter plots of the survived samples in the parameter space. The bullets (red) satisfy all considered constraints including $m_{\tilde{t}_1} < 1.5$ TeV and $m_{\tilde{t}_2} < 2$ TeV. The crosses (green) satisfy the constraints (1-4) but do not satisfy $m_{\tilde{t}_1} < 1.5$ TeV or $m_{\tilde{t}_2} < 2$ TeV. The two horizontal lines show the Higgs mass range of 123-127 GeV.

$m_{1/2}$ which further induces a light $\tilde{\tau}$. A light $\tilde{\tau}$ can lower the neutralino annihilation cross section through t -channel and thus raise the neutralino relic density.

At last, we show the fine-tuning extent versus the Higgs mass in Fig.3. Going beyond the tree-level expression (4.14), the loop-level minimization condition of the Higgs potential

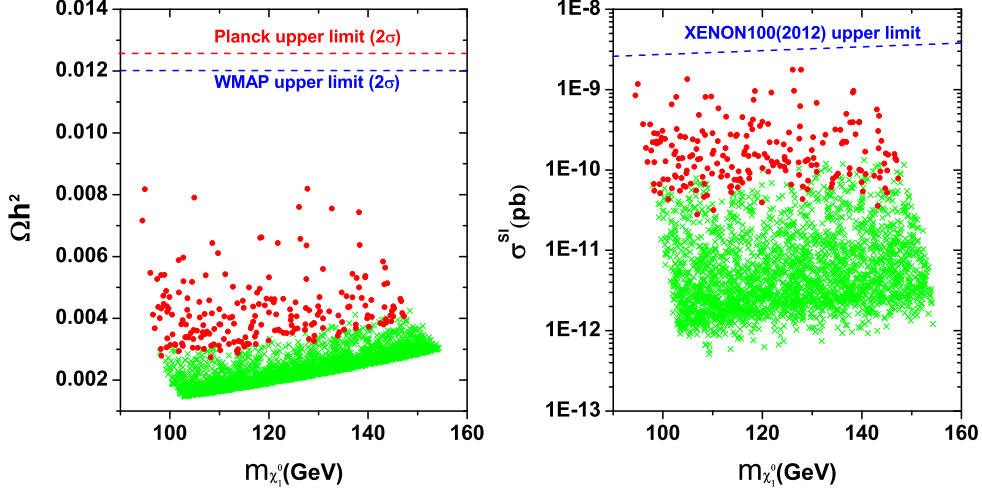


Figure 2: Same as Fig.1, but showing the neutralino relic density and the spin-independent neutralino-proton scattering cross section versus the neutralino mass. The horizontal lines show the 2σ upper limits from WMAP, Planck and XENON100(2012).

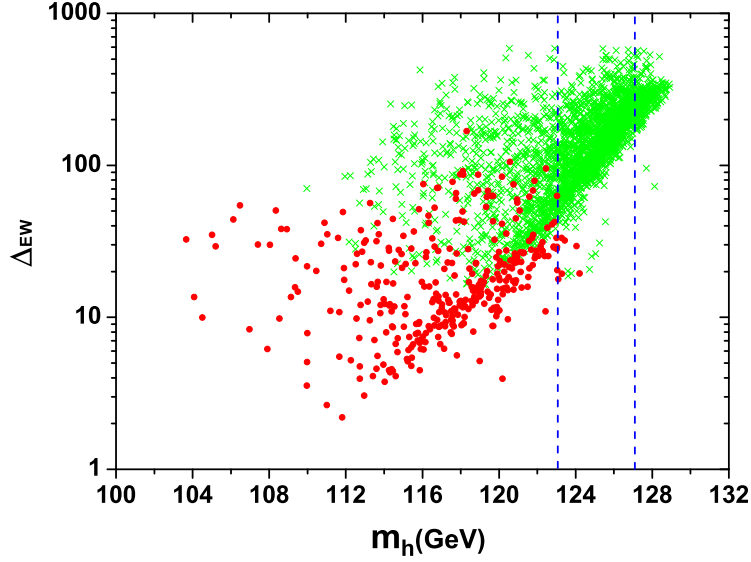


Figure 3: Same as Fig.1, but showing the fine-tuning extent versus the SM-like Higgs mass. The two vertical lines show the Higgs mass range of 123-127 GeV.

is

$$\frac{M_Z^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d) - (m_{H_u}^2 + \Sigma_u) \tan^2 \beta}{\tan^2 \beta - 1} - |\mu|^2, \quad (5.5)$$

where Σ_u and Σ_d are the radiative corrections to the Higgs potential and the dominant contribution to the Σ_u is given by

$$\Sigma_u \sim \frac{3Y_t^2}{16\pi^2} \times m_{\tilde{t}_i}^2 \left(\log \frac{m_{\tilde{t}_i}^2}{Q^2} - 1 \right). \quad (5.6)$$

The fine-tuning extent is defined by [42]

$$\Delta_{EW} = \frac{\max |C_i|}{M_Z^2/2}, \quad (5.7)$$

where $C_i (i = h_u, h_d, \mu, \Sigma_u, \Sigma_d)$ denotes each term in the right side of (5.5). From Fig.3 we see that in general light-stop points correspond to a low fine-tuning extent. Note that there are some heavy-stop points which can give a low fine-tuning extent (below 30). This means that in our scenario the radiative natural SUSY be realized.

6. Conclusions

In this paper, we proposed a realistic 5D orbifold GUT model that can reduce to (radiative) natural supersymmetry in the low energy. Supersymmetry as well as gauge symmetry are broken by the twist boundary conditions. We found that it is non-trivial to introduce other flavor symmetry other than the $SU(2)_R$ R-symmetry. The tension between the limited number of parameters and the successful electroweak symmetry breaking can be ameliorated by introducing non-minimal Kahler potentials. A large trilinear term A_t , which is necessary to give a 125 GeV higgs is naturally predicted in our scenario. We scan over the parameter space under current experiments and found that our model can realize (radiative) natural supersymmetry.

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